Optimal Design of Active Tuned Mass Dampers for Mitigating Translational–Torsional Motion of Irregular Buildings

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Abstract. The active tuned mass dampers (ATMDs) are accepted as effective energy dissipating devices to effectively reduce structural dynamic response when subjected to seismic loads. The conventional design of these devices, however, may not be applicable to high-rise buildings with irregularity in plan and elevation, where significant torsional motions could be dominant during a strong earthquake. This study is to develop a new approach using the ATMD for reducing the torsional motion as well as resisting the lateral translational displacements. Three actuators are used to apply the control forces to the twin-TMD system in two directions, while the optimal control forces were determined using linear quadratic regulator (LQR) algorithm. In addition, instead of using two independent mass dampers in two directions, a single damper system was used to minimize the displacements and rotation simultaneously. To demonstrate the performance of the system, the final design was applied to an irregular ten-story building subjected to near- and far-field earthquakes. The results indicate that the proposed design approach is more cost effective as compared to the design with independent pairs of dampers in two directions. Further, the system exhibits higher reliability under different ground accelerations in two directions than conventional ones.

Keywords: LQR control · Irregular building · Tuned mass damper (TMD) · Active tuned mass damper (ATMD) · Twin tuned mass damper (TTMD)

1 Introduction

During recent years, seismic response control of structures has gained a significant attention due to the life safety requirements and the tragic events related to major earthquakes [1, 2]. Reviewing the literature reveals the fact that several historic earthquakes such as 1940 El Centro, 1971 San Fernando, 1989 Loma Prieta, 1994 Northridge, 1995 Kobe, and 1999 Chi-Chi caused severe damages in civil structures

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[2–5]. Even though seismic provisions are revised each year based on the new findings in seismic performance design of structures, which may require existing building to be retrofitted according to the current design criteria [6], they do not guarantee the minimum damage due to the future unknown seismic events [7, 8]. Significant levels of damage to the structures with asymmetric design demands development of effective methods for protection of existing and new structures for a safer environment for humans [9].

The majority of researchers have concentrated on seismic performance of structures under unidirectional seismic loads using independent shear frame models in two directions; however, the fact is that earthquakes have indeed arbitrary direction, that can be described by a bidirectional excitation [10]. In addition, the nature of buildings is asymmetric, which cause torsion coupling (TC) vibration with simultaneous torsional–translational oscillation under bidirectional ground motion.

Bidirectional seismic loads can cause severe structural damage due to translation–torsion coupled vibrations, that is ignored in the shear frame models, and needs to be taken into consideration [11], particularly, for those structures with irregularity in plan and elevation [12, 13]. For instance, the axial forces in the corner columns are different when subjected to bidirectional or unidirectional ground motions [14]. Also, the border members—the transverse elements—can behave nonlinearly under bidirectional earthquakes [15].

The traditional approaches for reinforcing the structures tend to introduce additional stiffness and damping to the substandard structures [6]. Passive tuned mass dampers (TMD) are one of the attractive mass vibration absorbers for reducing the structural responses that consist of a mass, spring, and dashpot. TMDs have been employed in a large number of structures around the world [16, 17]. TMDs counteract the movement of floor when they are stretched or compressed. Although TMDs are theoretically high-performance devices, application of passive TMD systems are limited to a narrow frequency band and needs to be tuned according to the main structure. Therefore, they cannot be effective for such structures with closely spaced natural frequencies [18].

One of the variables in tuning a TMD is the mass ratio (μ) that is usually within the range of 1–10%. Using the mass ratio, μ, researchers used different optimal designs; the formulation for classic TMD designs are listed in [19]. However, the estimated natural frequency and mass can differ from the real ones due to estimation errors. Those effects will result in a suboptimal TMD tuning or even a mistuning, and thereby a loss of damping in the controlled structure [20].

Active-TMD and active mass driver (AMD) do not have the limitations of passive TMDs, they are considered as one of the most efficient control devices, due to their stability and high adaptability to seismic loading variations [21]. Such damping system requires a controller to drive the mass through an actuator. The design of a proper controller is as much important as the tuning for passive TMDs.

Although several proposals have been presented for designing TMD systems for 3D buildings under bi-directional earthquake loads, only limited studies have considered the torsional motion in their designs; therefore, an integrated system with optimum cost and performance needs to be developed. In this study, a new approach is presented for the optimal design of passive and active tuned mass dampers. The proposed twin tuned mass damper (TTMD) system is designed to reduce the translational movements of irregular buildings and also resist the torsional motion. The performance of the proposed TTMD and active-TTMD (ATTMD) is evaluated for a 10-story irregular building under bidirectional ground motions.

2 Optimal Vibration Control Using TTMD

2.1 Configuration of TTMD System

As shown in Fig. 1, the twin tuned mass damper (TTMD) system includes two masses, M_1 and M_2, that are connected to each other with a rigid bar; while the bar is restrained in the x-direction at midpoint, it can move in the y-direction and rotate about the midpoint O. Three springs and dashpots are used to stabilize the system. For the active-TTMD system, three actuators apply the controlling forces in two directions as well as the moment for suppressing the translational–torsional vibration. For simplicity, properties of each twin dampers are selected identical, and the optimal values were obtained through an optimization algorithm. The length of the connecting rigid bar is assumed to be 10 m.

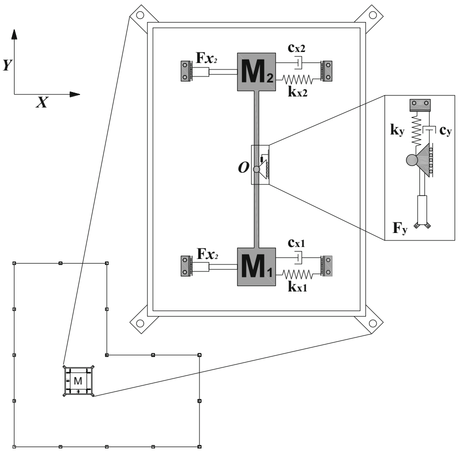

Fig. 1. Schematic diagram of the active twin tuned mass damper (ATTMD) installed on the roof level.

2.2 Optimal Design Using Particle Swarm Optimization (PSO) Algorithm

Intelligent algorithms are used extensively in structural engineering [22–25]. Since the optimization problem is continuous in this study, the particle swarm optimization (PSO) algorithm is used for the optimal design of TTMD parameters. The procedure is based on the PSO algorithm [26] that takes the parameters of the TTMD as the design variables and determines them to meet the objectives. The design objectives can be varied; in this study, in order to maintain the integrity of the building, the optimization problem is expressed as:

$$\begin{array}{l} \text{Find}: \ C_{x1,x2,y_{TTMD}}, K_{x1,x2,y_{TTMD}} \\ \text{Minimize}: \ X_{\max} = \max\left|x_{roof}(t)\right| \\ \text{Subjected to}: \ 0 < K < K_{\max}; 0 < C < C_{\max}; \end{array} \tag{1}$$

where C and K are the damping and stiffness of each dashpot and spring in both directions, respectively.

2.3 Governing Equations of the Controllable System Using TTMD and ATTMD

The governing equation of motion for a controlled nDOF system under seismic ground acceleration load, $\ddot{x}_g$, can be condensed and described as:

$$[\boldsymbol{M}]\{\ddot{x}(t)\} + [\boldsymbol{C}]\{\dot{x}\} + [\boldsymbol{K}]\{x\} = [\gamma]\{u(t)\} + \{\boldsymbol{\delta}\}\ddot{x}_g(t), \tag{2}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$, and $\boldsymbol{K}$ are ($3n \times 3n$) matrices of mass, damping, and stiffness, and $\{x\}, \{\dot{x}\}$, and $\{\ddot{x}\}$ are the ($n \times 1$) displacement, velocity and acceleration vectors, respectively and relative to the base of the building [27]. $\{u(t)\}$ is the control force vector, and $\{\boldsymbol{\delta}\}$ is the coefficient vector for earthquake ground acceleration $\ddot{x}_g(t)$. $[\gamma]$ is the controller location matrix that also represents the influence of each actuator on the other DOFs. Damping matrix of the system, C_{-s}, is determined using the Rayleigh method based on the mass and stiffness matrices [28, 29], and an inherent damping of 5% is assumed for the first and fourth modes of vibration. The equation of motion can be solved by rewriting it in state-space form as:

$$\left\{\dot{Z}(t)\right\} = [\boldsymbol{A}]\{Z(t)\} + [\boldsymbol{B_u}]\{u(t)\} + \{\boldsymbol{B_r}\}\ddot{x}_g(t), \tag{3}$$

where

$$\begin{array}{l} \{Z(t)\} = \begin{Bmatrix} x(t) \\ \dot{x}(t) \end{Bmatrix}; \quad \boldsymbol{A} = \begin{bmatrix} [\boldsymbol{0}] & \boldsymbol{I} \\ -\boldsymbol{M}^{-1}\boldsymbol{K} & -\boldsymbol{M}^{-1}\boldsymbol{C} \end{bmatrix}; \quad \{\boldsymbol{B_u}\} = \begin{bmatrix} [\boldsymbol{0}] \\ \boldsymbol{M}^{-1}[\gamma] \end{bmatrix}; \\ \{\boldsymbol{B_r}\} = \begin{bmatrix} \{\boldsymbol{0}\} \\ [\boldsymbol{M}]^{-1}\{\boldsymbol{\delta}\} \end{bmatrix}. \end{array} \tag{4}$$

The control forces for the active systems are calculated using the linear quadratic regulator (LQR). The LQR controller has recently gained significant importance, and it has been used for controlling the vibration of buildings with asymmetric plans in order to reduce the lateral-torsional responses [30].

2.4 Numerical Example: Ten-Story Irregular Steel Moment Frame

Inspired by the research carried out by Kaveh et al. [31], a ten-story irregular building frame was selected for numerical simulations. The 3D and plan views of the model are

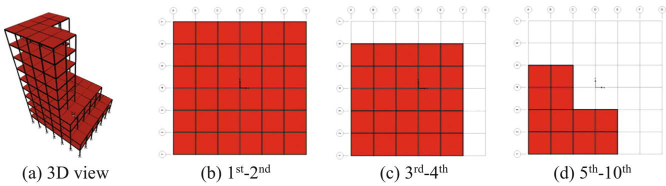

Fig. 2. 3D and plan views of the ten-story irregular building.

shown in Fig. 2. Following assumption are made in order to idealize the numerical model of the example: all the column members have the same symmetric cross section and height; each floor acts as a rigid diaphragm and all the columns are fixed at both ends. Therefore, the stiffness of each column in this model can be estimated as $k = 12EI/h^3$, where h is the story height, E is the modulus of elasticity of the steel material, and I is the moment of inertia in the x- and y-direction.

2.5 Modal Analysis

For an irregular building, higher modes of vibration may participate as well as the primary first modes during an earthquake. Since the model is neither symmetric nor has the same center of mass for all the stories, the modal analysis of the system was carried out in order to study the mode shapes (Fig. 3). From the mode shapes, it can be expected that the torsional motion of the building is inevitable under the bi-directional ground motions. Therefore, for such buildings with irregularity, it is necessary that the earthquake ground motion is applied in both directions simultaneously, and the influence of dimension, location and other properties of a TMD need to be considered in evaluating the performance of the system. In this study, the influence of the location of the TMD is not investigated; thus, it is assumed that it is located at the center of mass of the roof level.

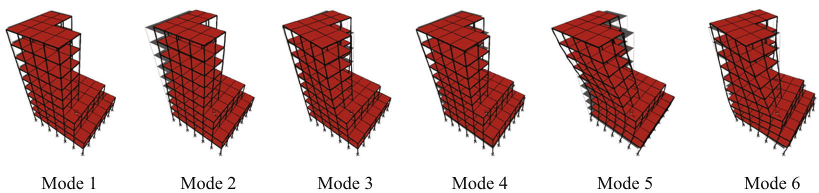

Fig. 3. Mode shapes of the building.

2.6 Optimization Problem Results

The optimum parameters for the typical TMD systems, for minimizing the roof displacement, can be obtained using the equations given in Table 1. The optimum design parameters of the traditional TMD system for different mass ratios can be obtained by using the equations in this table, which are basically developed for unidirectional earthquake excitation without considering the eccentricities. For three mass ratios of 1%, 3%, and 5%, the mass, damping, and stiffness of the TMD are given in Table 2.

Table 1. The optimal parameters for a TMD attached to a building [32].

Minimizing item	Frequency ratio	Damping ratio
Displacement	$\frac{1}{1+\mu}$	$\sqrt{\frac{3\mu}{8(1+\mu)^3}}$

Table 2. Optimum parameters of TMD using the traditional method (in one direction).

Criterion	μ	M_d (kg)	C_d (N/m/s)	K_d (N/m)
Displacement	1%	77,925	7763	53,119
	3%	233,775	37,662	147,336
	5%	389,625	75,761	227,379

The main concern about the traditional design of TMDs is that they do not consider the torsional motion of irregular buildings, however, they are proved to be effective for symmetric structures. Figure 4 shows the roof displacement of the 10-story building under the El Centro and Northridge earthquakes. It can be seen that by considering the torsional motion of the story levels, the maximum displacement at the corners of each floor are greater, which are neglected in the unidirectional design approach.

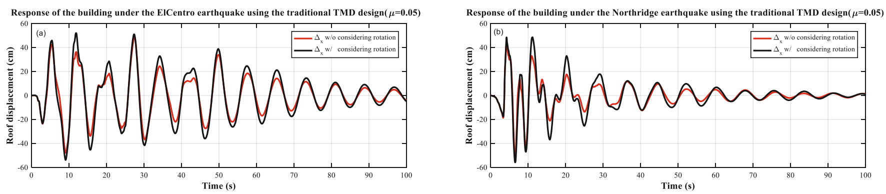

Fig. 4. Displacement response of the irregular building in x-direction with and without considering the torsional motion under the (a) El Centro and (b) Northridge earthquakes.

Therefore, the traditional approaches for the optimal design of TMD parameters for irregular buildings do not guarantee the superior performance of the system under bidirectional earthquake loads. In this study, the particle swarm optimization (PSO) algorithm is used in order to find the optimum design parameters of the proposed TTMD system. For this purpose, the 10-story irregular building is subjected to bidirectional random excitation that is obtained by filtering a white noise through a linear filter—known as Kanai–Tajimi filter—which represents the surface ground [19].

Figure 5 shows the number of PSO algorithm iterations vs. the best cost which is the displacement of the roof level in this study. Since irregular buildings have different configurations, it is preferable to find the optimum design parameters for the TTMD using the powerful metaheuristic algorithms. The procedure is repeated for three different mass ratios: μ = 1%, 3%, and 5%.

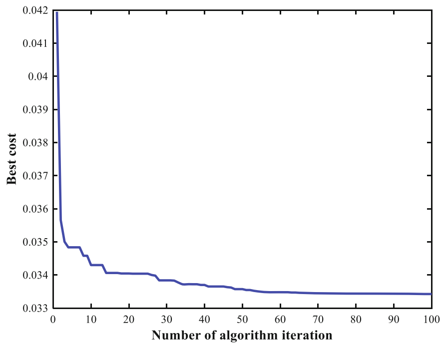

Fig. 5. The best design history of the controlled 10-story building with TTMD using PSO algorithm (μ = 0.05).

2.7 Performance of the Designed TTMD and ATTMD Under Historic Earthquakes

While the TTMD parameters are designed using the white noise excitation (Fig. 6), three historic earthquake records are selected in this research to evaluate the performance of the TTMD and ATTMD control systems under different ground excitations. The characteristics of the selected earthquakes are given in Table 3.

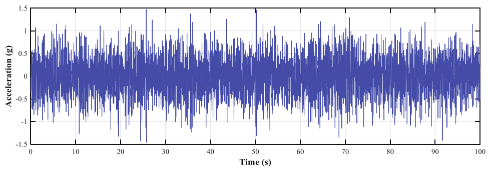

Fig. 6. Filtered white noise excitation.

Table 3. Characteristics of the historic earthquake records.

Earthquake*	Station and direction	Magnitude (M_w)	PGA (g)	PGV (cm/s)
1940 El Centro	El Centro Array #9 270°	7.2	0.21	30.2
	El Centro Array #9 180°	7.2	0.28	31.0
1994 Northridge	Sylmar - Olive View Med FF 360°	6.7	0.84	129.6
	Sylmar - Olive View Med FF 090°	6.7	0.61	77.53
1995 Kobe	H1170546.KOB 090°	7.2	0.63	76.6
	H1170546.KOB 000°	7.2	0.83	91.13

*Source: http://ngawest2.berkeley.edu/

The torsional motion of the building is considered in the proposed control system; for the proposed ATTMD, the actuators generate a moment in addition to the forces in two directions. The moment counteracts the torsional motion; therefore, the overall displacements in both directions, particularly for those columns in the corners, are reduced. Figure 7 shows the maximum displacement response of the building at the roof level—the corner columns—as well as the roof rotation using the three control systems, TMD, TTMD, and ATTMD. It is evident that by using the proposed design approach for TTMD system, both translational and torsional responses of the irregular building is reduced significantly compared to the traditional TMD system with the same mass ratios. In addition, the active form of the TTMD system (ATTMD) considerably reduces the response of the building during the excitation time. For example, from the displacement responses, by using the ATTMD control system there is no noticeable displacement after 50th second of the ground excitations. The maximum responses for each scenario are given in Table 4. In this table, $x_{\max}$ and θ_{roof} are the maximum displacement and rotation at the roof level, respectively.

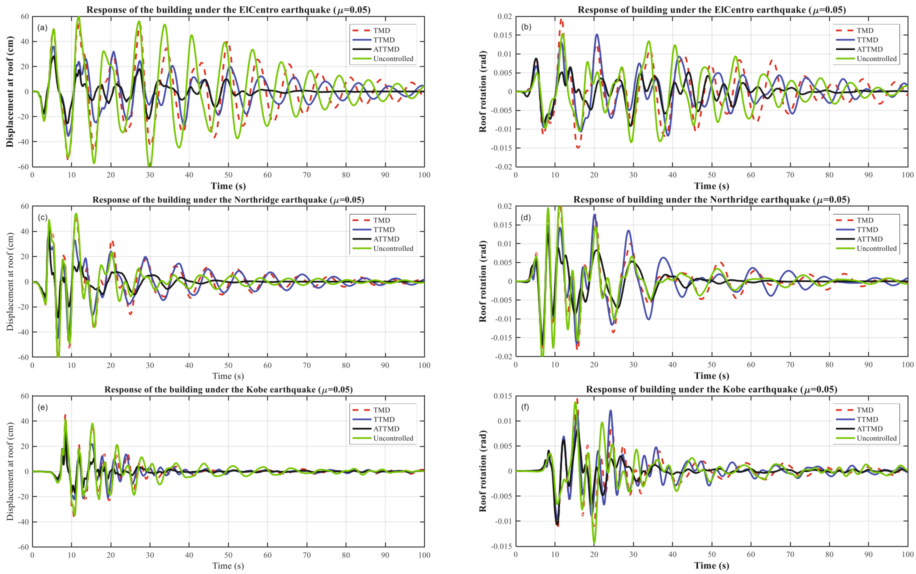

Fig. 7. Displacement and rotation responses at roof level under the El Centro, Northridge, and Kobe earthquakes.

From Table 4, it can be seen that the traditional optimal design does not guarantee the maximum reduction of response in the irregular 10-story building. For example, by using the TMD system with 5% mass ratio, the maximum reduction in the displacement response obtained under the El Centro earthquake (21%), however, the same design increases the displacement and the rotation at roof level by 7% and 1%, respectively. On the other hand, the proposed TTMD system offers significant response reduction for all the mass ratios and under all three earthquake records. The maximum response

Table 4. Maximum responses of the uncontrolled building and the response reduction percentage for each control technique.

μ	Earthquake	Response	Uncontrolled	TMD	TTMD	ATTMD
1%	El Centro	x_{max}	69.4 cm	−7%	−32%	**−65%**
		θ_{roof}	0.0210 rad	−1%	−25%	**−44%**
	Northridge	x_{max}	72.3 cm	−5%	−30%	−45%
		θ_{roof}	0.0204 rad	−1%	−25%	−11%
	Kobe	x_{max}	42.4 cm	**+7%**	−16%	−23%
		θ_{roof}	0.0145 rad	**+1%**	−19%	−26%
3%	El Centro	x_{max}	69.4 cm	−16%	−44%	**−60%**
		θ_{roof}	0.0210 rad	−4%	−33%	**−48%**
	Northridge	x_{max}	72.3 cm	−7%	−30%	−44%
		θ_{roof}	0.0204 rad	−1%	−25%	−13%
	Kobe	x_{max}	42.4 cm	**+7%**	−16%	−26%
		θ_{roof}	0.0145 rad	**+0%**	−20%	−26%
5%	El Centro	x_{max}	69.4 cm	**−21%**	**−48%**	**−59%**
		θ_{roof}	0.0210 rad	**−6%**	**−40%**	**−56%**
	Northridge	x_{max}	72.3 cm	−8%	−38%	−53%
		θ_{roof}	0.0204 rad	−1%	−28%	−31%
	Kobe	x_{max}	42.4 cm	**+7%**	−19%	−25%
		θ_{roof}	0.0145 rad	**+1%**	−31%	−37%

reduction is for the TTMD design with 5% mass ratio under El Centro earthquake by 48%. Although for the building controlled by TMD, greater mass ratios result in more reduction, for the active-TTMD, the maximum reduction in the response is achieved using the least mass ratio, μ = 1%. Differences between the reduction percentages suggest that it is more appropriate to optimize the design case-by-case and separately, and avoid using set of equations for all configurations. Particle swarm optimization (PSO) algorithm is a powerful tool in finding the optimal design parameters of the proposed TTMD for buildings with irregularities in plan and elevation.

3 Conclusions

A new structural control approach is introduced in designing the active tuned mass dampers (ATMDs) for irregular high-rise buildings. The optimal parameters of the proposed twin tuned mass damper (TTMD) systems are obtained through the particle swarm optimization (PSO) algorithm using a filtered white noise as the ground acceleration. The performance of the proposed system is evaluated for a 10-story irregular building under three historic earthquakes that are applied bi-directionally to the system. The results confirm that, despite the traditionally designed TMDs, the proposed TTMD is more effective in reducing the translational–torsional motion of irregular structures. In addition, the active-TTMD (ATTMD) control system is also investigated in this study, which significantly decreases the responses by using the linear quadratic regulator (LQR) for determining the optimal control forces. The results of this study may lead to introduce new design criteria for designing of tuned mass damper systems for irregular buildings.

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